Assign 4 Q 4

**Hypothesis**: reduce and reduce\_tr will return the same result given the same arguments, for any chosen set of arguments.

**AXIOMS**

**Axiom A:**

op(base, m) = m

**Axiom B:**

op(n,m) = op(m,n)

**Axiom C:**

op(n, op(m,k))) = op(op(n,m), k)

**LEMMA**

**Lemma:** op(h, reduce(l, n, op)) = reduce\_tr(l, op(h, n), op)

**Proof by structural induction on l:**

**Base Case:**

for any h, op, and n, let l = []

op(h, reduce([], n, op)) = reduce\_tr([], op(h, n), op)

|-> op(h, n) = op(h, n)

**Induction Hypothesis:**

Op (h, reduce (l, n, op)) = reduce\_tr (l, op (h, n), op)

**Proof:**

Show that op (h, reduce (a::l, n op)) = reduce\_tr (a::l, op (h, n), op)

op (h, reduce (a::l, n op)) = reduce\_tr (a::l, op (h, n), op)

by definition of reduce and reduce\_tr:

|-> op (h, op (a, reduce (l, n, op))) = reduce\_tr (l, op (a, op (h, n)), op)

By axioms B and C:

|-> op (op (h, a), reduce (l, n, op)) = reduce\_tr (l, op (op (h, a), n), op)

By induction hypothesis

|-> op (op (h, a), reduce (l, n, op)) = op (op (h, a), reduce (l, n, op))

**PROOF**

**Proof by structural induction on a:**

**Base Case:**

For any base and op, let a = []

reduce ([], base, op) = reduce\_tr ([], base, op)

(fun reduce ([], base, op) = base) = (fun reduce\_tr ([], base, op) = base)

Base = base

**Induction Hypothesis:**

Reduce (a, base, op) = reduce\_tr (a, base, op)

**Proof:**

Show that reduce (b::a, base, op) = reduce\_tr (b::a, base op)

By definition of reduce and reduce\_tr

|-> op(b, reduce(a, base, op)) = reduce\_tr (a, op(b, base), op)

By Lemma

|-> op(b, reduce(a, base, op)) = op(b, reduce(a, base, op))

This concludes the proof.